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RELIABILITY ANALYSIS OF REINFORCED CONCRETE COLUMNS USING THE GENERAL AND EQUILIBRIUM METHODS

ANÁLISE DA CONFIABILIDADE DE PILARES DE CONCRETO ARMADO ATRAVÉS DOS MÉTODOS GERAL E DO EQUILÍBRIO

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Abstract
This paper aims to present a structural reliability analysis of reinforced concrete columns through the General and Equilibrium Methods, being the probabilistic analysis developed through the First Order Reliability Method (FORM) and validated through Monte Carlo simulation. The study was developed from an algorithm that enables the verification of reinforced concrete columns, up to a maximum slenderness index equal to 200, subject to bi-axial bending and compression, considering the physical and geometric nonlinearities according to the standard recommendations of NBR 6118 (2014). The random variables considered are actions (dead and live), strength of materials (steel and concrete), section geometry, and modelling error. The results showed that the most important parameters in the probability of failure are the slenderness index and the load ratio. Finally, the results allowed for a parametric analysis of the variables involved in the problem, making it possible to evaluate the sensitivity of each parameter on the system reliability.
Keywords: Reliability Analysis, Slender RC columns, General Method, Equilibrium Method.

Resumo
Este trabalho tem como objetivo apresentar uma análise da confiabilidade estrutural de pilares de concreto armado através dos Métodos Geral e do Equilíbrio, sendo a análise probabilística desenvolvida através do Método de Confiabilidade de Primeira Ordem (FORM), que foi validada através da simulação de Monte Carlo. O estudo foi desenvolvido a partir de um algoritmo que possibilita a verificação de pilares de concreto armado, até um índice de esbeltez máximo igual a 200, sujeitos à flexo-compressão oblíqua, levando-se em conta as não-linearidades física e geométrica de acordo com as recomendações normativas da NBR 6118 (2014). As variáveis aleatórias consideradas são as ações (permanente e variável), resistência dos materiais (aço e concreto), geometria da seção e erro do modelo. Os resultados demostraram que os parâmetros de maior importância na probabilidade de falha são o índice de esbeltez e a razão de carga. Por fim, os resultados possibilitaram uma análise paramétrica das variáveis envolvidas no problema, sendo possível avaliar a sensibilidade de cada parâmetro na confiabilidade do sistema.
Palavras-chave: Análise de confiabilidade, Pilares esbeltos de CA, Método Geral, Método do equilíbrio.

1. INTRODUCTION

Among the national studies on reliability of reinforced concrete columns are Nogueira (2006), who evaluated the reliability of short columns; Deuschle (2019), who studied reliability considering the strength redistribution; and Araújo (2001), Damas (2015), Barbosa (2017), and Oliveira (2018), who developed their analyses using the Finite Element Method. Barbosa (2017) and Damas (2015) evaluated the reliability of columns designed according to the Approximate Stiffness and Approximate Curvature Methods, therefore, the studies were limited to a slenderness index up to a maximum of 90. Similarly, Oliveira (2018) presented as limit the slenderness index equal to 90, evaluating the reliability obtained by simplified design methodologies.

Internationally, Szerszen, Szwed, and Nowak (2005) and Kim et al. (2015) have evaluated reliability based on interaction diagrams; and Hong and Zhou (1999) have used a simplistic approach that considers the correlation between axial load and bending moment, which does not require the use of simulation methods. In addition to these mentioned works, there are others involving the subject of reliability in reinforced concrete columns, such as Mirza and MacGregor (1989), Ruiz and Aguilar (1994), Frangopol et al. (1996) and Mirza (1996). It should be noted that these studies represent the deformed axis of the column through simplified expressions.

In this paper, the reliability of reinforced concrete columns under second order effects will be investigated through the General and Equilibrium Methods, designed according to the recommendations of NBR 6118 (2014). The First Order Reliability Method (FORM) and the Importance Sampling Monte Carlo (ISMC) were employed. From the simulations performed, the most important parameters in the probability of failure are the slenderness index and the ratio between the live and dead loads.

2. NBR 6118 (2014) CRITERIA

NBR 6118 (2014) allows the use of four different methods for analyzing the second-order effects on columns, described in the following.

- Standard column method with approximate curvature: It is employable on columns with $\lambda \leq 90$, with constant section and symmetrical and constant reinforcement along the height. Both geometric and physical nonlinearity are considered roughly. It is based on the idealization that the bar deformation is sine and nonlinearity can be considered through an approximate expression in the critical section;
- Standard column method with approximate κ stiffness: It is allowed on columns with $\lambda \leq 90$, with constant section and symmetrical and constant reinforcement along the height. It is based on the idealization that bar deformation is sinusoidal and physical nonlinearity can be considered through an approximate expression of stiffness;
- Standard Column Method coupled to normal force and bending moment diagrams: It is a coupling of the diagrams to the standard column method or improved standard column, allowed to use in columns with $\lambda \leq 140$ provided that the values obtained in case-specific M, N, 1/r diagrams are used for bending of the critical section;
- General Method: Mandatory for columns with $\lambda > 140$, in which the actual moment-curvature relationship in each section is considered, and geometric nonlinearity is considered in a non-simplified way.

2.1. Remarks on the design methodologies

As the General Method requires the solution of differential equations that generally do not present explicit solutions, the calculation is performed by iterative methods with approximate solutions. Essentially, it is studied the behavior as loading or eccentricity increases.

As a summary, Table 1 presents recommendations regarding the applicability of each method, as well as its validity domain, highlighting that the General Method is applicable in any situation. Table 1 shows that from λ =90 it becomes mandatory to consider the creep effects and not allow the application of simplified methodologies, unless the diagrams for bending-axial force-curvature are coupled.

Table 1. NBR 6118 (2014) recommendations for columns regarding the methodologies of calculation.

<table>
<tr><th rowspan="2">Slenderness Index</th><th rowspan="2">Disregard second-order local effects</th><th rowspan="2">Disregard creep effects</th><th rowspan="2">γ_f</th><th colspan="3">Simplified Methodologies</th><th rowspan="2">General Method</th></tr>
<tr><th>Standard column with approximate curvature</th><th>Standard column with dimensional stiffness</th><th>Standard column coupled to diagrams N, M, 1/r</th></tr>
<tr><td>$\lambda \leq \lambda_1$
(low slenderness)</td><td>A</td><td rowspan="2">A</td><td rowspan="3">1.4</td><td rowspan="2">A</td><td rowspan="2">A</td><td rowspan="3">A</td><td rowspan="4">A</td></tr>
<tr><td>$0 < \lambda \leq 90$
(average slenderness)</td><td rowspan="3">NA</td></tr>
<tr><td>$90 < \lambda \leq 140$
(slender)</td><td rowspan="2">NA</td><td rowspan="2">NA</td><td rowspan="2">NA</td></tr>
<tr><td>$140 < \lambda \leq 200$
(high slenderness)</td><td>(1)</td><td>NA</td></tr>
<tr><td colspan="8">A: Allowed; NA: Not Allowed.
(1): $1.4 + 0.01(\lambda - 140)$.
λ_1 is defined by NBR 6118 (2014) and depends on the first-order eccentricity, the column connection, and the shape of the stress diagram.</td></tr>
</table>

2.2. Ultimate Limit State

As a failure criterion, for the evaluation of the limit state function, two criteria are used, namely, evaluation to know if there was failure at the section level (reaching the Ultimate Equilibrium Limit State), or if the failure occurs by instability characterizing the Instability Limit State. Ultimate Limit State as defined by NBR 6118 (2014) is related to collapse, or any other form of structure ruin. Breakage at the section level can occur through two forms: excessive plastic deformation of steel and/or crushing by shortening of concrete. From the combination of possible deformation states in a section, the deformed condition will be classified into any of the deformation domains that can be: line a, 1, 2, 3, 4, 4a, 5, and line b. The possible domains in a bending-axial compression ranges from 2 to 5, with the eccentricity of loading being the main parameter involved in the determination of the failure domain.

In the Ultimate Equilibrium Limit State, NBR 6118 (2014) admits the following hypotheses for linear elements subject to normal demands: (*i*) Application of the Navier-Bernoulli hypothesis; (*ii*) The deformation in the steel bars is the same as the concrete in its surroundings (adhesion); (*iii*) Disregard of concrete tensile strength; (*iv*) Admitted that the maximum elongation in the tensile reinforcement is equal to 10 ‰; (*v*) The distribution of stresses in concrete is made according to the idealized parabola-rectangle stress-strain diagram.

The stress-strain diagram idealized for concrete is presented in Figure 1, being applied for the analysis of the long-term effects, the creep coefficient (φ) of the Linear Creep Theory. Figure 1 also shows the idealized stress-strain diagram of steel with well-defined plastic flow level.

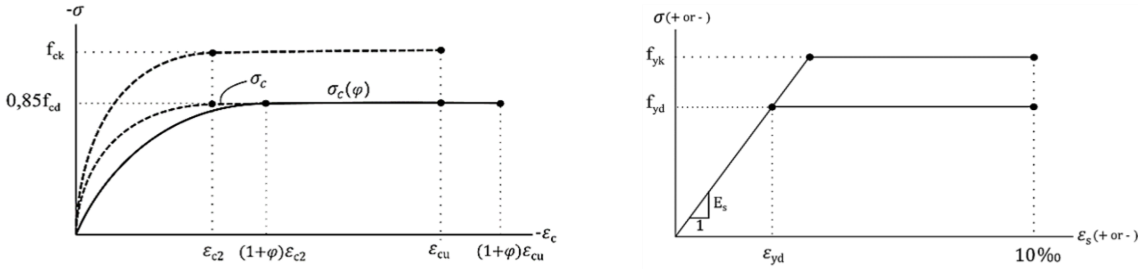


Figure 1. Material behavior: concrete (left) and steel (right).

For concrete class up to C50, the values of ε_{c2} and ε_{cu} are equal to 2 ‰ and 3.5 ‰, respectively. For the concretes of group II (above 50 MPa), the values of ε_{c2} and ε_{cu} are obtained from Equations (1) and (2), respectively.

$$\varepsilon_{c2} = 2.0‰ + 0.085‰(f_{ck} - 50)^{0.53} \tag{1}$$

where f_{ck} is the characteristic compressive concrete strength.

$$\varepsilon_{cu} = 2.6‰ + 35‰\left(\frac{90 - f_{ck}}{100}\right)^{4} \tag{2}$$

From the experimentation, it can be observed that under the action of increasing loadings, the axially compressed straight bars reach a limit state, called instability in axial compression, in which the straight form of equilibrium is unstable, and the stable form of equilibrium in the elastic regime is a bending configuration. The load corresponding to that limit state is called a critical load or buckling load. If the bar is analyzed above the proportionality limit, that is, there is no longer the linear elasticity of the material, for a load above the critical value, the change of the equilibrium form corresponds to an unstable behavior and the bending shape is impossible. Moreover, for structural materials (steel and concrete) instability is assumed an Ultimate Limit State because the bar presents considerable displacements for loads slightly higher than the critical, therefore, resulting in rupture by bending (Fusco 1981).

2.3. Equilibrium Method

This method, in conjunction with the General Method, checks whether the column is stable without having to determine its critical load. In this way, it is verified if for a given load the horizontal deflection of the sections of the discretization employed results in a stable situation. The process iteratively calculates the (second order) loads along the column from its deformed configuration so that it is possible to determine the corresponding curvatures and finally obtain the displacements by integrating the curvatures.

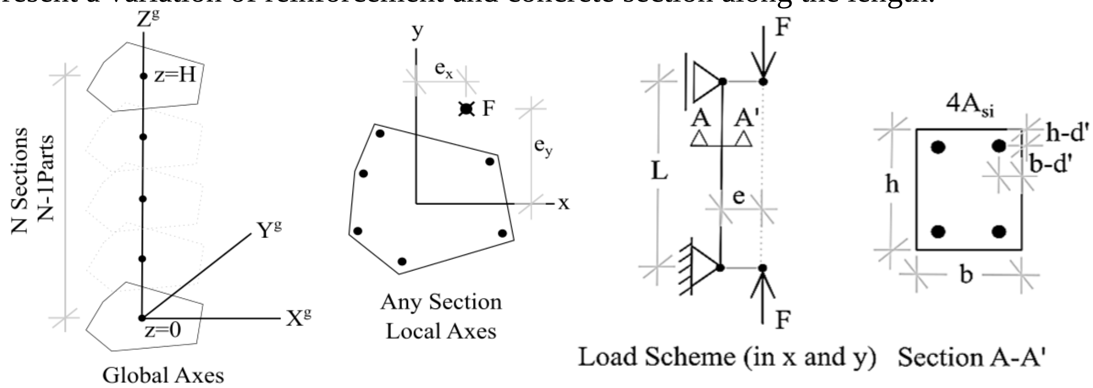
3. METHODOLOGY: ALGORITHM EMPLOYED

The algorithm used for the evaluation of the limit state function, in the case of the present article an Ultimate Limit State for collapse, is presented in Cadamuro Junior (1997), therefore, it is recommended for reading of this work for a complete and detailed explanation of the algorithm. This algorithm enables the design and verification of reinforced concrete columns with any polygonal section loaded by compression and biaxial bending, considering physical and geometric nonlinearities, applying the Equilibrium Method with the General Method, as recommended by NBR 6118 (2014).

In this approach of using the General Method, the curvatures, horizontal displacements and second order moments of the discretization sections are calculated. Furthermore, as a simplifying assumption, it is assumed that the curvature varies linearly between two consecutive sections.

The determination of transverse displacements is done through a double numerical integration of the differential equation of curvature along the length of the column, being the rotations obtained at the first integration. This procedure allows the calculated displacements to present a cubic variation. With the horizontal displacements, the second order bending moments are calculated, which will be the main data needed for the iterative process.

Figure 2(a) shows how a column is discretized in order to use the algorithm. The column is discretized into N sections along its height. Each section can have a polygonal cross-section shape or any reinforcement arrangement and can even have internal voids in the polygon. It can also present a variation of reinforcement and concrete section along the length.



(a) (b)

Figure 2. (a) Illustration of an algorithm discretization. (b) Geometry of the studied column.

4. PROBABILISTIC ANALYSIS

In a reliability analysis, a limit state function $g(\boldsymbol{X})$ must be defined that represents the failure condition. In this work, this function was described as the difference between the rupture load (which presents the possibility of failure due to balance or instability) and the active loading, as presented in Equation (3). In addition, in this formulation there is the application of the modelling error, which aims to statistically represent the uncertainty of the modelling assumptions.

$$g(\boldsymbol{X}) = R(\boldsymbol{X}) \cdot E_{mod} - S(\boldsymbol{X}) \tag{3}$$

where: $R(\boldsymbol{X})$ is the rupture load, which configures some of the possible Ultimate Limit State, obtained using an algorithm such as bisection. This load is normal and eccentric with respect to the two axes that may cause the collapse, with the normal load and bending moment perfectly correlated; E_{mod} is the modelling error, estimated to be equal to 0.967; $S(\boldsymbol{X})$ is the loading, being composed of the sum of the live and dead loads in each simulation performed.

The random variables used for the evaluation of the limit state function are declared in Table 2. The design load used in the reliability analysis was determined by traditional column design with the use of partial safety coefficients, which were: (*i*) Coefficient for decrease in concrete strength $\gamma_c = 1.4$; (*ii*) Coefficient for decrease in Steel resistance $\gamma_s = 1.15$; (*iii*) Coefficient of increase of loads $\gamma_f = 1.4$, and in cases of slenderness index greater than 140, it is adopted the expression presented in Table 1.

The column system studied is shown in Figure 2(b). It is simply supported column, in both axes, presenting biaxial bending-compression characterized by an eccentric load. The cross section with the nominal design values has a base and height of the section equal to 30 cm, reinforcement clear height equal to 27 cm, with 4 bars of 20 mm diameter.

The probabilistic analysis was performed using the FORM, which was initially validated by the ISMC runs, that is, the same values were obtained with the two techniques in the tests performed. Thus, it was chosen FORM due to low computational cost involved in solving problems with relative accuracy.

Table 2. Statistical properties of random variables.

Variable	**Probability density function**	**Mean value (μ)**	**Coefficient of variation (V)**	**Standard deviation (σ)**
Compressive strength of concrete (f_c)	Normal	$\mu_{fc} = \dfrac{f_{ck}}{1 - 1.65 \cdot V_{fc}}$	0.10	$\sigma_{fc} = \mu_{fc} \cdot V_{fc}$
Steel yielding strength (f_y)	Log-normal	$\mu_{fy} = 1.09 \cdot f_{yk}$	0.05	$\sigma_{fy} = \mu_{fy} \cdot 0.05$
Dimensions of the cross section (d, b, h)	Normal	Nominal design value	σ/μ	0.5
Dead load (G)	Normal	Characteristic value of the action	0.10	$\mu_G \cdot 0.10$
Live Load (Q)	Gumbel	Characteristic value of the action	0.25	$\mu_Q \cdot 0.25$
Modelling error (E_{mod})	Normal	0.967	0.038	0.037

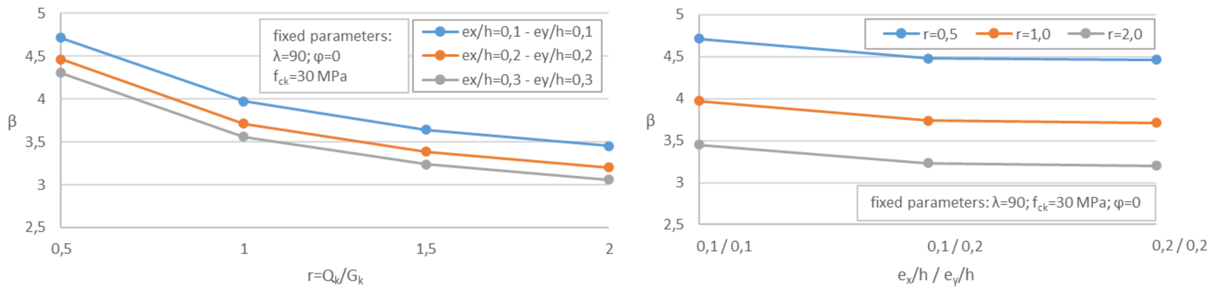
5. PARAMETRIC RESULTS AND ANALYSIS

105 different cases of combination of input parameters were analyzed to perform the parametric analyses of the columns subject to oblique bending-compression. For this, the distributions indicated in Table 2 were used, in addition to the reference section already presented. For the modulus of elasticity of the steel, the deterministic value of 210 GPa was used. Only in the reliability analysis for different reinforcement rates the standard reinforcement of the article was not employed. In this case, full percentage reinforcement ratios were used, even if they did not correspond to commercial measurements.

Figure 3(a) shows the effect of increasing the load ratio on reliability. It can be observed that as the load ratio increases, reliability decreases, because the accidental portion of the load, which has the highest coefficient of variation, becomes the predominant load. In Figure 3(b), it is shown how the increase in first order eccentricity influences reliability. It can be observed that the decrease in reliability was greater from the first to the second criterion evaluated than between the second and the third, because a larger eccentricity reduces the design load more significantly.

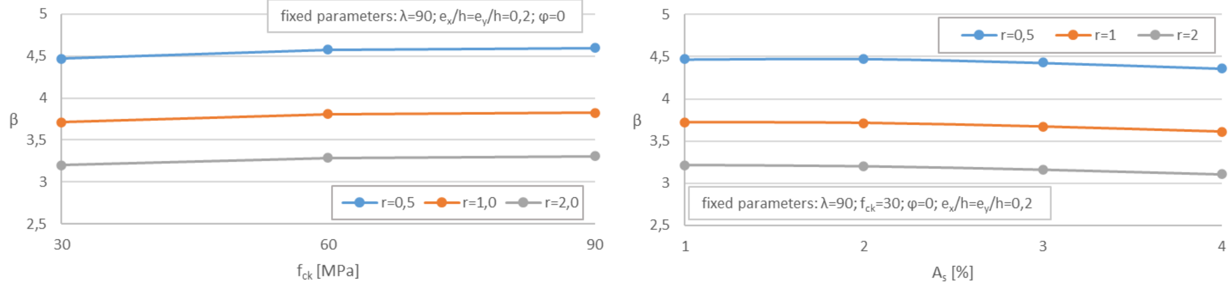
Figure 4(a) shows the effect of increasing concrete compressive strength on reliability. It can be observed that the high-performance concretes presented a higher reliability in the simulations performed. This difference is observed because concretes above C50 present a more conservative design procedure. In Figure 4(b), it is shown how the increase in the reinforcement ratio influences the reliability. It can be observed that as the ratio increases, the reliability index decreases. This effect occurs because in sections with high reinforcement ratios the design load is greater, and consequently, accidental loading becomes greater, resulting in more violations of the limit state function.

Finally, Figure 5 presents the evaluation of the effect of slenderness on reliability, evaluating different configurations of creep, load ratio and slenderness. It can be observed that, in general, creep did not significantly influence the reliability results. On the other hand, the load ratio, as in all analyses, proved to be very important. With respect to the slenderness index, it is observed that the results decrease until an index equal to 120, and soon after, presents an opposite behavior, of increase. This opposite behavior is explained by the fact that an adjustment in the coefficient (γ_f) is employed, according to the expression presented in Table 1, which causes an increase in reliability.



(a) (b)

Figure 3. Reliability results for load ratio (a) and first-order relative eccentricity values (b).



(a) (b)

Figure 4. Reliability results for concrete strength (a) and reinforcement rate values (b).

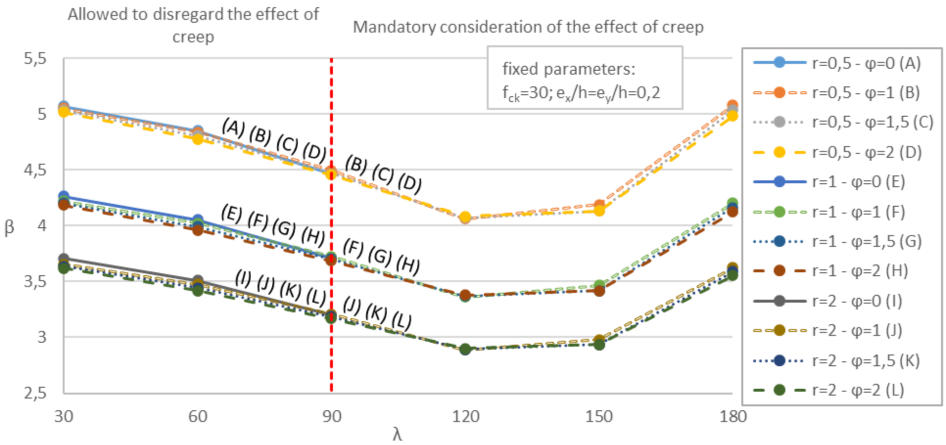


Figure 5. Reliability results for several slenderness index.

6. FINAL REMARKS

In this paper, an analysis of the reliability achieved in the design of reinforced concrete columns following the recommendations of NBR 6118 (2014) from the General Method in conjunction with the Equilibrium Method through FORM was presented.

It has been shown that reliability depends on several factor, such as the design load, the relative first-order eccentricity, the slenderness index, the dimensional variabilities of the column, as well as load variability and materials properties.

Some results were presented for excessively slender columns, little investigated so far, which showed that the design procedures are not very conservative for situations where the slenderness index is greater than 60 and less than 120.

The random variables that showed the highest sensitivity in the analyses (with a director cosine greater than 0.3 in most simulations) were the characteristics compressive strength of

concrete, accidental loading, and model error, with the variable load showing the most importance in most of the cases studied.

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REFERENCES

Araújo, J.M. (2001). A confiabilidade no projeto de pilares de concreto armado. Teoria e Prática na Engenharia Civil, n. 2, p. 1-8.

Associação Brasileira de Normas Técnicas (2014). Projeto de estruturas de concreto - procedimento. NBR 6118-2014. Rio de Janeiro.

Barbosa, P.R.O. (2017). Análise probabilística de pilares de concreto armado através do método dos elementos finitos. M.Sc. Thesis, UFRGS, PPGEC, Porto Alegre.

Cadamuro Júnior, I.W. (1997). Dimensionamento de pilares esbeltos de concreto armado com seção qualquer solicitados por flexão composta oblíqua. M.Sc. Thesis, USP, PPGSET, São Carlos.

Damas, A.P. (2015). Estudo de confiabilidade no projeto de pilares esbeltos de concreto de alta resistência. M.Sc. Thesis, UFRGS, PPGEC, Porto Alegre.

Deuschle, E. (2019). Estudo de confiabilidade de pilares de concreto armado dimensionados com redistribuição de esforços. M.Sc. Thesis, UFSC, PPGEC, Florianópolis.

Frangopol, D.M.; Ide, Y.; Spacone, E.; Iwaki, I. (1996). A new look at reliability of reinforced concrete columns. Structural Safety, v. 18, p. 123-50.

Fusco, P.B. (1981). Estruturas de concreto: solicitações normais. Editora Guanabara Dois, Rio de Janeiro, RJ, Brasil.

Hong, H.P. and Zhou, W. (1999). Reliability evaluation of RC columns. Journal of Structural Engineering, v. 125, p. 784-90.

Kim, J.H.; Lee, S.H.; Paik, I.; Lee, H.S. (2015). Reliability assessment of reinforced concrete columns based on the P-M interaction diagram using AFOSM. Structural Safety, v. 55, p. 70-79.

Mirza, S.A. (1996). Reliability-based design of reinforced concrete columns. Structural Safety, v. 18, p. 179-94.

Mirza, S.A. and MacGregor, J.G. (1989). Slenderness and strength reliability of reinforced columns. ACI Structural Journal, v. 86, p. 428-38.

Nogueira, H.A.T. (2006). Avaliação da confiabilidade de pilares curtos em concreto armado projetados Segundo a NBR 6118:2003. M.Sc. Thesis, UFMG, PROPEEs, Belo Horizonte.

Oliveira, A.H.C.R. (2018). Estudo comparativo da confiabilidade de pilares de concreto armado dimensionados de acordo com a norma brasileira NBR 6118:2014 e com o código americano ACI 318:2014. M.Sc. Thesis, UFRGS, PPGEC, Porto Alegre.

Ruiz, S.E. and Aguilar, J.C. (1994). Reliability of short and slender reinforced-concrete columns. Journal of Structural Engineering, ASCE, v. 120, p. 1850-65.

Szerszen, M.M.; Szwed, A.; Nowak, A.S. (2005). Reliability analysis for eccentrically loaded columns. ACI Structural Journal, v. 102, p. 676-88.